

Holographic Viscosity of Fundamental Matter

David Mateos,¹ Robert C. Myers,^{2,3,4} and Rowan M. Thomson,^{3,4}

¹ *Department of Physics, University of California, Santa Barbara, CA 93106-9530, USA*

² *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, USA*

³ *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*

⁴ *Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada*

A holographic dual of a finite-temperature $SU(N_c)$ gauge theory with a small number of flavours $N_f \ll N_c$ typically contains D-branes in a black hole background. By considering the backreaction of the branes, we demonstrate that, to leading order in N_f/N_c , the viscosity to entropy ratio in these theories saturates the conjectured universal bound $\eta/s \geq 1/4\pi$. The contribution of the fundamental matter η_{fund} is therefore enhanced at strong 't Hooft coupling λ ; for example, $\eta_{\text{fund}} \sim \lambda N_c N_f T^3$ in four dimensions. Other transport coefficients are analogously enhanced. These results hold with or without a baryon number chemical potential.

PACS numbers:

Introduction: A universal bound was recently proposed [1] for the ratio of the shear viscosity to the entropy density of any physical system as $\eta/s \geq 1/4\pi$. In particular, this bound is conjectured to hold for all relativistic quantum field theories at finite temperature that exhibit hydrodynamic behaviour at long wave-lengths. Perhaps surprisingly, experimental results from the Relativistic Heavy Ion Collider (RHIC) suggest that, for QCD just above the deconfinement phase transition, the value of η/s is close to saturating this bound [2]. This would indicate that the quark-gluon plasma formed at RHIC is an almost perfect liquid.

Unfortunately, at present there are no theoretical tools with which to calculate transport coefficients in QCD in this strong coupling regime, *e.g.*, the viscosity. However, a large class of gauge theories are accessible to study with the gauge/gravity correspondence [3]. In particular, in the gauge theory limit of large N_c and large 't Hooft coupling λ , the dual description reduces to classical supergravity. In fact, the proposal [1] for a universal bound on η/s originated with calculations in this holographic context. Explicit calculations [4, 5, 6, 7] and general arguments [1, 8, 9] have demonstrated that the bound is exactly saturated by a large class of holographic theories in the limit cited above. In order to make contact with real-world QCD, it is clearly important to consider $1/\lambda$ and $1/N_c$ corrections. For four-dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM), the leading correction of the first type was shown to raise the value of η/s above the bound [10]. That is, at finite coupling the bound still holds but is no longer saturated.

A feature common to all of the gauge theories considered in these hydrodynamic studies is that the matter degrees of freedom transform in the adjoint representation of the gauge group [11]. In this letter, we study the effect of adding matter fields transforming in the fundamental representation, bringing us one step closer to QCD. In particular, we focus on four-dimensional $SU(N_c)$ SYM coupled to N_f fundamental hypermultiplets with $N_f \ll N_c$. Large- N_c counting rules imply that, in the deconfined phase, the contribution of the gluons

and adjoint matter to physical quantities is of order N_c^2 . Further, the first correction in the absence of fundamental matter is of order 1, *i.e.*, the relative contribution is suppressed by $1/N_c^2$. Instead, the relative contribution of fundamental matter is only suppressed by N_f/N_c , and therefore it constitutes the leading correction in the large- N_c limit.

The dual gravity description is given by N_f D7-brane probes [13] in the background geometry of N_c D3-branes. At finite temperature, the latter contains a black hole [14]. We will show that, to leading order in N_f/N_c , the calculation of the η/s ratio can be effectively reduced to one in five-dimensional Einstein gravity coupled to a scalar field. General results [1, 8] then guarantee that $\eta/s = 1/4\pi$. Since the D7-brane contribution to the entropy density is known to be of order $s_{\text{fund}} \sim \lambda N_c N_f T^3$ [15], this implies that the contribution of the fundamental matter to the shear viscosity at strong 't Hooft coupling is enhanced with respect to that dictated solely by large- N_c counting rules.

In the final section we will argue that an analogous enhancement takes place for other transport coefficients. We will also explain how our results extend straightforwardly to other holographic gauge theories described by Dp/Dq [15, 16] or $Dp/Dq/D\bar{q}$ [17] systems, as well as to systems with a non-zero baryon number chemical potential. We will finish with some comments on effects beyond order N_f/N_c .

Holographic Framework: The shear viscosity of the gauge theory in a two-plane labelled by x^i, x^j may be extracted from the retarded correlator of two stress energy tensors via Kubo's formula

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{ij}(x), T_{ij}(0)] \rangle, \quad (1)$$

where no summation over i, j is implied. The stress energy tensor is dual on the string side to a metric perturbation H_{ij} polarised along the same two-plane. The two-point function above may be calculated by taking two functional derivatives of the on-shell string effective action with respect to this perturbation [18]. In the large-

N_c , large- λ limit, this effective action reduces to the type IIB supergravity action coupled to the worldvolume action of the D7-branes, $I = I_{\text{IIB}} + I_{\text{D7}}$. Schematically, we have:

$$I = \frac{1}{16\pi G} \int d^{10}x \sqrt{-g} R - N_f T_{\text{D7}} \int d^8x \sqrt{-g_{\text{ind}}} + \dots, \quad (2)$$

where g_{ind} is the induced metric on the D7-branes. In terms of the string length and coupling:

$$16\pi G = (2\pi)^7 \ell_s^8 g_s^2, \quad T_{\text{D7}} = 2\pi / (2\pi \ell_s)^8 g_s. \quad (3)$$

The ratio between the normalisations of the two terms above is

$$\varepsilon = 16\pi G N_f T_{\text{D7}} = \frac{\lambda}{2\pi} \frac{N_f}{N_c}, \quad (4)$$

where $\lambda = g_{\text{YM}}^2 N_c = 2\pi g_s N_c$ is the 't Hooft coupling. This ratio controls the relative magnitude of the D7-branes' contribution to physical quantities, *e.g.*, the entropy density [15]. We will assume that $\varepsilon \ll 1$ and hence that the D7-branes can be treated as a small perturbation; for fixed λ this is achieved by taking $N_f \ll N_c$. We will begin by examining contributions of order ε in the next section. In the last section, we will comment on effects of order ε^2 and higher.

In the absence of D7-branes, the supergravity background dual to four-dimensional $\mathcal{N} = 4$ SYM at temperature T is (in the notation of [15])

$$ds^2 = ds_5^2 + L^2 d\Omega_5^2, \quad (5)$$

$$ds_5^2 = \frac{(\pi L T \rho)^2}{2} \left[-\frac{f^2}{\tilde{f}} dt^2 + \tilde{f} dx_i^2 \right] + \frac{L^2}{\rho^2} d\rho^2, \quad (6)$$

where

$$f(\rho) = 1 - 1/\rho^4, \quad \tilde{f}(\rho) = 1 + 1/\rho^4, \quad (7)$$

and $L = (4\pi g_s N_c)^{1/4} \ell_s$ is the asymptotic AdS radius. There are also N_c units of Ramond-Ramond (RR) flux through the five-sphere while the remaining supergravity fields vanish. The metric (5,6) possesses an event horizon at $\rho = 1$. The entropy density of the gauge theory is then given by the geometric entropy of the horizon [19]

$$s = \frac{\pi^3}{4G_5} L^3 T^3 = \frac{\pi^2}{2} N_c^2 T^3, \quad (8)$$

where $G_5 = G/\pi^3 L^5$ is the five-dimensional Newton's constant obtained by dimensional reduction on the five-sphere.

Now we introduce the D7-branes oriented such that five worldvolume directions match those of the five-dimensional black hole (6), $y \equiv \{t, x^i, \rho\}$, and the remainder wrap an S^3 (with a possibly varying radius) inside the S^5 of (5). We adapt coordinates in this internal space such that

$$d\Omega_5^2 = d\theta^2 + \sin^2 \theta d\Omega_3^2 + \cos^2 \theta d\phi^2 \quad (9)$$

and describe the D7-branes embedding as $\chi = \chi(\rho)$, with $\chi = \cos \theta$. To order ε^0 , this is determined by extremising the D7-brane action in the background (5,6). Asymptotically, one finds

$$\chi = \frac{m}{\rho} + \frac{c}{\rho^3} + \dots, \quad (10)$$

where m and c are proportional to the quark mass M_q and condensate $\langle \bar{\psi}\psi \rangle$, respectively. In the interior, the D7-branes may or may not reach the black hole horizon [15].

Viscous Branes: As alluded to above, the calculation of the shear viscosity proceeds as follows. First, one solves the (linearised) equation of motion for a metric perturbation H around the appropriate background. Next, one evaluates the appropriate action for the perturbed background to quadratic order in H . A second derivative of the on-shell action then yields the desired two-point function [18] with which the Kubo formula (1) is evaluated.

In the absence of D7-branes the appropriate action is I_{IIB} and the background is given by (5). In the presence of the D7-branes the relevant action is instead that in eq. (2). To first order in the ε -expansion, this affects the calculation of the viscosity in three ways. First, the branes will produce $\mathcal{O}(\varepsilon)$ corrections to the metric (5), as well as to the dilaton and the RR axion, since they act as new sources in the field equations arising from the combined action. These background corrections then lead to modifications of the field equation satisfied by H . Second, the branes will also modify the H field equation directly through the extra source terms originating from the variation of I_{D7} with respect to H . Third, the second-derivative of the on-shell action, which yields the correlator in Kubo's formula, may acquire contributions from I_{D7} . [20]

We will now show that only the first two types of possible modifications do actually contribute and, moreover, that the only type of background correction that needs to be considered is the zero-mode (on the five-sphere) of the five-dimensional black hole metric (6).

We begin by considering the third set of possible contributions listed above. Expanding the brane action around the $\mathcal{O}(\varepsilon^0)$ background, one finds that, because H enters the action non-derivatively, the H^2 terms do not have a form which will contribute in the Kubo formula [18]. However, turning on H also induces a correction $\delta\chi = \mathcal{O}(H^2)$ in the embedding of the branes. This leads to a surface term in the variation of the D7-brane action,

$$\delta I_{\text{D7}} \sim \frac{\partial \mathcal{L}_{\text{D7}}}{\partial (\partial_\rho \chi)} \delta \chi \Big|_{\rho_{\text{max}}}, \quad (11)$$

of the right form to contribute to the two-point correlator. However, this contribution is proportional to δm [15] and hence vanishes because the variation of the action with respect to H must be taken while keeping the quark mass fixed.

Consider now corrections to the background (5). While there are $\mathcal{O}(\varepsilon)$ corrections to the dilaton and the RR axion, these only produce $\mathcal{O}(\varepsilon^2)$ contributions to the H field equation because they enter the supergravity action quadratically (in the Einstein frame). We are thus left to consider the contributions of corrections to the spacetime metric. Schematically, to order ε , the background metric takes the form

$$g = g_0(y) + \varepsilon g_1^{(0)}(y) + \varepsilon \sum_{\ell \neq 0} g_1^{(\ell)}(y) Y^{(\ell)}(\Omega), \quad (12)$$

where $Y^{(\ell)}(\Omega)$ are spherical harmonics on S^5 . The zero'th order metric (5) and the correction $g_1^{(0)}$ are constant on the S^5 . However, since the D7-branes only fill an S^3 in the internal space, they also source the $g_1^{(\ell)}$ corrections which vary over the S^5 . For the following, an important point is that the functions $g_1^{(0)}(y)$ and $g_1^{(\ell)}(y)$ respect the symmetries of the background geometry (5) and the brane embedding, *i.e.*, translations in $\{t, x^i\}$ and $SO(3)$ rotations in x^i , as well as $SO(4)$ rotations in the internal S^3 wrapped by the D7-brane. The perturbation H has a similar decomposition:

$$H = H_0(y) + \varepsilon H_1^{(0)}(y) + \varepsilon \sum_{\ell \neq 0} H_1^{(\ell)}(y) Y^{(\ell)}(\Omega). \quad (13)$$

In the absence of D7-branes, one works consistently with the S^5 -independent perturbation H_0 [4, 5]. However, in the presence of the D7-branes, nontrivial $H_1^{(\ell)}$ are sourced when H_0 is turned on. Indeed, after integration over the S^5 , the supergravity action produces couplings of the schematic form $\varepsilon^2 \int d^5y H_0 H_1^{(\ell)} g_1^{(\ell)}$. Similarly, the D7-branes action produces couplings like $\varepsilon^2 \int d^5y H_0 H_1^{(\ell)}$, which arise from spherical harmonics $Y^{(\ell)}$ that are constant on the S^3 wrapped by the D7-branes. However, as indicated, both types of terms are of order ε^2 and so we may neglect their contribution since here we only wish to determine the correlator (1) up to order ε .

We therefore conclude that, to order ε , we need only consider the zero-modes $g_1^{(0)}(y)$ and $H_1^{(0)}(y)$, and may drop all modes with non-trivial dependence on the S^5 directions. Hence in working to order ε , evaluation of the viscosity actually reduces to a five-dimensional calculation. We can make the latter concrete by dimensionally reducing the action (2) to five dimensions ignoring all the Kaluza-Klein modes on the five-sphere, as well as the other supergravity fields. By the previous arguments, the resulting action still captures all of the relevant fields for the calculation of the viscosity to this order. The five-dimensional action can be written as

$$I_5 = \frac{1}{16\pi G_5} \int d^5y \sqrt{-g} \left[R + \frac{12}{L^2} - \frac{2\varepsilon}{\pi L^2} (1 - \chi^2) \times \sqrt{1 - \chi^2 + L^2 g^{\rho\rho} (\partial_\rho \chi)^2} \right], \quad (14)$$

where g denotes the five-dimensional metric. The first two terms originate from the reduction of I_{IIB} , whereas

the last one comes from the reduction of I_{D7} [15]. Note that, in the action (14), we have only allowed scalar field configurations depending on the radial coordinate, since this suffices for our purposes. This system is just five-dimensional Einstein gravity coupled to a cosmological constant and a(n unusual) scalar field χ . In an ε -expansion, the black hole solutions generated by this auxiliary theory will match the asymptotically AdS part of the original ten-dimensional solution to order ε , *i.e.*, the brane profile $\chi(\rho)$ and the background metric (6) plus the order- ε correction $g_1^{(0)}(\rho)$.

The viscosity may now be obtained by calculating the perturbation H_{ij} around the five-dimensional solution and taking the second functional derivative of the action (14) evaluated on-shell. However, the black hole solutions of our auxiliary five-dimensional system satisfy the symmetries required in [1], and hence the result is guaranteed to satisfy $\eta/s = 1/4\pi$. We thus conclude that this universal bound is still saturated in the full ten-dimensional string theory when working to first order in ε . An immediate consequence is that the contributions of the fundamental matter to the viscosity and the entropy density are related (within our approximations) as $\eta_{\text{fund}} = s_{\text{fund}}/4\pi$. The leading contribution to the entropy density was determined [15] to be

$$s_{\text{fund}} = \frac{\lambda}{16} N_c N_f T^3 h\left(\frac{\lambda T}{M_q}\right), \quad (15)$$

where the function $h(x)$ satisfies $h(0) = 0$, $h(\infty) = 1$, and makes a cross-over between both values around $x \sim 1$. Note that this cross-over includes a small discontinuity arising from a first-order phase transition of the fundamental matter [15]. We therefore conclude that both s_{fund} and η_{fund} are enhanced at strong 't Hooft coupling with respect to the $\mathcal{O}(N_c N_f)$ -value dictated solely by large- N_c counting rules.

The calculation of s_{fund} in [15] was performed by identifying the Euclidean action of the D7-branes with F_{fund}/T , where F_{fund} is the free energy contribution of the fundamental matter. The entropy is then determined as $s_{\text{fund}} = -\partial F_{\text{fund}}/\partial T$. This entropy should, of course, coincide with the change in the horizon area induced by the presence of the D7-branes. The latter can be explicitly verified for the case of massless quarks, which corresponds to an 'equatorial embedding' with $\chi = 0$. In this case, the result from [15] for the entropy density is $s_{\text{fund}} = \lambda N_c N_f T^3/16$. In the action (14), we see that the net effect of these 'equatorial' D7-branes is to shift the effective cosmological constant. The corresponding black hole solution is still given by (6), with the replacement $L^2 \rightarrow L^2/(1 - \varepsilon/6\pi)$. The same replacement in (8) shifts the entropy to order ε by $\delta s = \lambda N_c N_f T^3/16$, in perfect agreement with the previous result.

Discussion: We have seen that the calculation of the contribution of fundamental matter to the shear viscosity may be effectively reduced to a calculation in five dimensions. An analogous simplification takes place for other

transport coefficients that can be extracted from correlators involving local operators with vanishing R-charge, since these are dual to modes that carry no angular momentum on the S^5 . Examples involving components of the stress-energy tensor include the speed of sound v_s and the bulk viscosity ξ . Other transport coefficients that involve R-charged operators, such as the R-charge diffusion constant [5], or extended strings, such as the jet quenching parameter \hat{q} (see *e.g.*, [21]), may require a ten-dimensional calculation. Generically, however, we expect the relative contribution of the fundamental matter to be of order $\varepsilon \sim \lambda N_f/N_c$, since this controls the backreaction of the branes. A special case is the bulk viscosity, which may be extracted from a two-point function of $T_{ii} - v_s^2 T_{00}$ [22]. This combination vanishes if $N_f = 0$ by conformal invariance, and hence $\xi = \mathcal{O}(\varepsilon^2)$ in the presence of fundamental matter.

Above, our discussion focussed on the D3/D7 system, but the arguments are easily extended to a more general system of Dq-branes in a Dp-background. In particular, systems arising from Dp- and Dq-branes intersecting over d common spatial directions have been of some interest [16, 23]. These constructions are dual to a finite-temperature SYM theory in $p + 1$ dimensions coupled to fundamental matter confined to a $(d + 1)$ -dimensional defect. One new feature in these generalised configurations is that the defect breaks translational invariance along the $p - d$ orthogonal directions. In order to calculate the shear viscosity [24] along the translationally invariant directions parallel to the defect, the simplest approach is to compactify these extra directions [26]. The arguments in the previous section go through essentially unchanged except for the fact that the index ℓ now labels momentum modes both along the S^{8-p} transverse to the Dp-branes and along the $p - d$ directions orthogonal to the defect. In this case the problem of calculating the leading contribution of the fundamental matter to the viscosity/entropy ratio can be reduced to a calculation in $(d + 2)$ -dimensional Einstein gravity coupled to a set of scalar fields. In addition to the scalar χ above describing the size of the internal S^n of the Dq-brane, this set now includes the dilaton and the metric components governing the size of the internal S^{8-p} of the background geometry and the size of the $(p - d)$ -dimensional space orthogonal to the defect [27]. This lower dimensional theory again captures all of the relevant fields to calculate the viscosity to leading order in N_f/N_c . Further, the form of the $(d + 2)$ -dimensional gravity theory and the background guarantees that $\eta/s = 1/4\pi$. The leading result for the entropy density was determined in [15] and hence we have

$$\eta_{\text{fund}} \sim N_c N_f T^d g_{\text{eff}}^{\frac{2(d-1)}{5-p}}(T), \quad (16)$$

where $g_{\text{eff}}^2(T) = \lambda T^{p-3}$ is the dimensionless effective 't Hooft coupling for a $(p + 1)$ -dimensional theory at temperature T [28]. Here the gauge/gravity duality is only valid in the strongly coupled regime [28] and hence we see again an enhancement beyond the large- N_c counting.

It would be interesting to understand if this enhancement extends to other transport properties in the same way, as was found for the D3/D7 case. The same line of argument can also be implemented for the Dp/Dq/D \bar{q} systems which have also been studied recently [17].

The $U(N_f) \simeq SU(N_f) \times U(1)_B$ gauge symmetry on the Dq-branes is a global, flavour symmetry of the dual gauge theory. The results above also hold when a baryon number chemical potential for the $U(1)_B$ charge is introduced. This is dual [29] to turning on the time component of the gauge potential on the Dq-branes, $A_0(\rho)$ [30]. The arguments of the previous section again go through essentially unchanged, except for the fact that an additional vector A_μ is added to the reduced $(d + 2)$ -dimensional Einstein gravity theory. Thus the saturation of the bound is not affected by the introduction of a chemical potential.

Above we have worked to the lowest order in the parameter $\varepsilon \sim \lambda N_f/N_c$ that controls the backreaction of the D7-branes on the D3-brane geometry. We have argued that to this order one may ignore all effects of this backreaction except for those on the non-compact part of the metric. We regard the agreement between the entropy density as calculated in [15] and as obtained here from the change in the horizon area as a quantitative consistency check of this approach. In calculating beyond order ε , the internal modes, *e.g.*, $g_1^{(\ell)}(y)$, and other supergravity fields, *e.g.*, the RR axion, will all play a role. Further at $\mathcal{O}(\varepsilon^2)$, quantum effects will have to be considered [31]. Closed string loop corrections naturally appear in an expansion in $g_s^2 \sim \lambda^2/N_c^2$. Thus if as above N_f is fixed, the loop corrections may be of the same magnitude as the higher order D7-brane contributions, *i.e.*, $g_s^2 \sim \varepsilon^2$ if $N_f = \mathcal{O}(1)$.

In order to suppress string loop corrections with respect to the backreaction of the D7-branes (or more generally the Dq-branes), one must keep N_f/N_c fixed while taking $N_c \rightarrow \infty$. In this limit, the entire backreaction of the branes must be taken into account. A fully backreacted solution is singular for the D3/D7 system [32], since the dual gauge theory possesses a Landau pole at some finite scale Λ_{UV} . This of course should not affect the hydrodynamic behaviour as long as the other scales in the problem, the temperature and the quark mass, are much lower than Λ_{UV} (just like the transport properties of an electromagnetic plasma are not affected by the Landau pole in QED). Similarly, in the case of a general Dp-brane background (with $p \neq 3$), one finds strong coupling or curvature divergences in the far UV, which are irrelevant for the long-wavelength hydrodynamics. In these cases, the classical backreacted solution would effectively resum the effects of the fundamental matter to leading order in the $1/\lambda, 1/N_c$ expansions. It may be possible to prove that the viscosity bound is still saturated by these solutions by extending the arguments of [8, 9]. The finite-temperature backgrounds of [12, 33] represent a step towards this goal.

Acknowledgments: We thank H. Elvang, S. Mathur, S. Shenker, and very especially R. Emparan for useful

discussions. We also thank A. Starinets for his collaboration at an early stage. DM thanks the Aspen Center for Physics and the KITP for hospitality during the various stages of this project. Research at the Perimeter Institute is supported in part by funds from NSERC of Canada

and MEDT of Ontario. We also acknowledge support from NSF grant PHY-0244764 (DM), NSERC Discovery grant (RCM) and NSERC Canada Graduate Scholarship (RMT). Research at the KITP was supported in part by the NSF under Grant No. PHY99-07949.

-
- [1] P. Kovtun, D.T. Son and A.O. Starinets, JHEP **0310**, 064 (2003) [arXiv:hep-th/0309213]; Phys. Rev. Lett. **94**, 111601 (2005) [arXiv:hep-th/0405231];
 - [2] E. Shuryak, Prog. Part. Nucl. Phys. **53**, 273 (2004) [arXiv:hep-ph/0312227]; D. Teaney, Phys. Rev. C **68**, 034913 (2003) [arXiv:nucl-th/0301099].
 - [3] J.M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
 - [4] G. Policastro, D.T. Son and A.O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001) [arXiv:hep-th/0104066].
 - [5] G. Policastro, D.T. Son and A.O. Starinets, JHEP **0209** (2002) 043 [arXiv:hep-th/0205052].
 - [6] C.P. Herzog, JHEP **0212**, 026 (2002) [arXiv:hep-th/0210126]; A. Buchel, Nucl. Phys. B **708**, 451 (2005) [arXiv:hep-th/0406200]; P. Benincasa and A. Buchel, Phys. Lett. B **640**, 108 (2006) [arXiv:hep-th/0605076].
 - [7] J. Mas, JHEP **0603**, 016 (2006) [arXiv:hep-th/0601144]; D.T. Son and A.O. Starinets, JHEP **0603**, 052 (2006) [arXiv:hep-th/0601157]; O. Saremi, arXiv:hep-th/0601159; K. Maeda, M. Natsuume and T. Okamura, Phys. Rev. D **73**, 066013 (2006) [arXiv:hep-th/0602010]; A. Buchel and J.T. Liu, arXiv:hep-th/0608002.
 - [8] A. Buchel and J.T. Liu, Phys. Rev. Lett. **93**, 090602 (2004) [arXiv:hep-th/0311175]; A. Buchel, Phys. Lett. B **609**, 392 (2005) [arXiv:hep-th/0408095].
 - [9] P. Benincasa, A. Buchel and R. Naryshkin, arXiv:hep-th/0610145.
 - [10] A. Buchel, J.T. Liu and A.O. Starinets, Nucl. Phys. B **707**, 56 (2005) [arXiv:hep-th/0406264]; P. Benincasa and A. Buchel, JHEP **0601**, 103 (2006) [arXiv:hep-th/0510041].
 - [11] An exception is ref. [12], where fundamental matter was considered. However, the construction involves D5-branes and so the temperature is not a free parameter.
 - [12] R. Casero, C. Nunez and A. Paredes, Phys. Rev. D **73**, 086005 (2006) [arXiv:hep-th/0602027].
 - [13] A. Karch and L. Randall, JHEP **0106**, 063 (2001) [arXiv:hep-th/0105132]; A. Karch and E. Katz, JHEP **0206**, 043 (2002) [arXiv:hep-th/0205236].
 - [14] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].
 - [15] D. Mateos, R.C. Myers and R.M. Thomson, Phys. Rev. Lett. **97**, 091601 (2006) [arXiv:hep-th/0605046]; “Thermodynamics of the brane”, to appear.
 - [16] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270].
 - [17] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141]; **114**, 1083 (2006) [arXiv:hep-th/0507073]; O. Aharony, J. Sonnenschein and S. Yankielowicz, arXiv:hep-th/0604161; N. Horigome and Y. Tanii, arXiv:hep-th/0608198; E. Antonyan, J.A. Harvey and D. Kutasov, arXiv:hep-th/0608177; arXiv:hep-th/0608149; arXiv:hep-th/0604017; A. Parnachev and D.A. Sahakyan, Phys. Rev. Lett. **97**, 111601 (2006) [arXiv:hep-th/0604173]; Y. Gao, W. Xu and D. Zeng, JHEP **0608**, 018 (2006) [arXiv:hep-th/0605138]; D. Gepner and S. Sekahr Pal, arXiv:hep-th/0608229; A. Basu and A. Maharana, arXiv:hep-th/0610087.
 - [18] D.T. Son and A.O. Starinets, JHEP **0209**, 042 (2002) [arXiv:hep-th/0205051]; C.P. Herzog and D.T. Son, JHEP **0303**, 046 (2003) [arXiv:hep-th/0212072].
 - [19] S.S. Gubser, I.R. Klebanov and A.W. Peet, Phys. Rev. D **54**, 3915 (1996) [arXiv:hep-th/9602135].
 - [20] At higher orders there are of course other contributions. For example, at $\mathcal{O}(\varepsilon^2)$ there is a contribution due to the fact that background corrections modify the D7-brane embedding.
 - [21] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L.G. Yaffe, JHEP **0607**, 013 (2006) [arXiv:hep-th/0605158]; H. Liu, K. Rajagopal and U.A. Wiedemann, arXiv:hep-ph/0605178; S. S. Gubser, arXiv:hep-th/0605182; J. Casalderrey-Solana and D. Teaney, arXiv:hep-ph/0605199.
 - [22] A. Hosoya, M. Sakagami and M. Takao, Annals Phys. **154**, 229 (1984).
 - [23] D. Arean and A.V. Ramallo, JHEP **0604**, 037 (2006) [arXiv:hep-th/0602174]; R.C. Myers and R.M. Thomson, JHEP **0609**, 066 (2006) [arXiv:hep-th/0605017]; D. Arean, A.V. Ramallo and D. Rodriguez-Gomez, arXiv:hep-th/0609010.
 - [24] The framework for the calculation of correlators is less well developed for such backgrounds, which are not asymptotically AdS. Cascading gauge theories are an interesting case where these techniques are being developed [25].
 - [25] M. Krasnitz, arXiv:hep-th/0011179; JHEP **0212**, 048 (2002) [arXiv:hep-th/0209163]; O. Aharony, A. Buchel and A. Yarom, Phys. Rev. D **72**, 066003 (2005) [arXiv:hep-th/0506002]; arXiv:hep-th/0608209; M. Berg, M. Haack and W. Muck, Nucl. Phys. B **736**, 82 (2006) [arXiv:hep-th/0507285]; A. Buchel, Phys. Rev. D **72**, 106002 (2005) [arXiv:hep-th/0509083].
 - [26] Compactifying is also required to make the $(d+2)$ -dimensional Newton’s constant finite. Since Newton’s constant cancels in the ratio η/s , there is no obstruction to taking the infinite volume limit at the end.
 - [27] One might similarly worry about the RR field sourced by the Dq-branes. This will be of order ε and so naturally contributes to the stress tensor at order ε^2 . However, there may also be order- ε cross-terms if this RR field already appears as part of the background generated by the Dp-brane. For supersymmetric cases [23], this only happens for the D3/D3, D2/D4 and D1/D5 systems. In all of these, the defect is $(1+1)$ -dimensional and so the

shear viscosity is not defined.

- [28] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, Phys. Rev. D **58**, 046004 (1998) [arXiv:hep-th/9802042].
- [29] N. Horigome and Y. Tanii, arXiv:hep-th/0608198; S. Kobayashi, D. Mateos, S. Matsuura, R.C. Myers and R.M. Thomson, to appear.
- [30] This baryon number chemical potential should not be confused with the R-charge chemical potential considered in *e.g.* [7, 9], which is dual in the string description to angular momentum on the sphere.
- [31] We expect Hawking radiation contributes at order N_f^2/N_c^2 . In the presence of N_f D7-branes, there are (at least) $\mathcal{O}(N_f^2)$ species or degrees of freedom into which the black hole can Hawking radiate. However, this is still suppressed by $1/\lambda^2$ relative to ε^2 at strong 't Hooft coupling.
- [32] I. Kirsch and D. Vaman, Phys. Rev. D **72**, 026007 (2005) [arXiv:hep-th/0505164]; B.A. Burrington, J.T. Liu, L.A. Pando Zayas and D. Vaman, JHEP **0502**, 022 (2005) [arXiv:hep-th/0406207].
- [33] M. Gomez-Reino, S. Naculich and H. Schnitzer, Nucl. Phys. B **713**, 263 (2005) [arXiv:hep-th/0412015].